Your Signature ____________________

This is a closed book exam. Show all your work. Correct answers with insufficient or incorrect work will not get any credit. Maximum possible score is 80. There are four questions.

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1. (20 points) Write a `mytriag` function that returns the upper triangular entries of the matrix if the input is a matrix and if the input is a vector returns a diagonal matrix with the entries being the vector. (*No credit for using inbuilt functions such as* `triau`.)

2. (25 points)Solve the following questions.

   (a) Define (and list) the terms: logical operators and relational operators.

   (b) Describe the output of the following OCTAVE commands
   `>> x = -12:12 .`
   Then use the `linspace` function to create the same.

   (c) Describe the output of the following OCTAVE commands
   ```
   >> x = [0 5 3 7 ];
   >> y = [0 2 8 7];
   >> u = x( (x>y) & (x>4))
   ```

   (d) Describe how a real number would be stored as a single precision floating point number.

   (e) Define what is meant by the term `flops`. Estimate upto "big O" the flops required for multiplication of $v_{n\times 1}$ and $A_{n\times n}$.

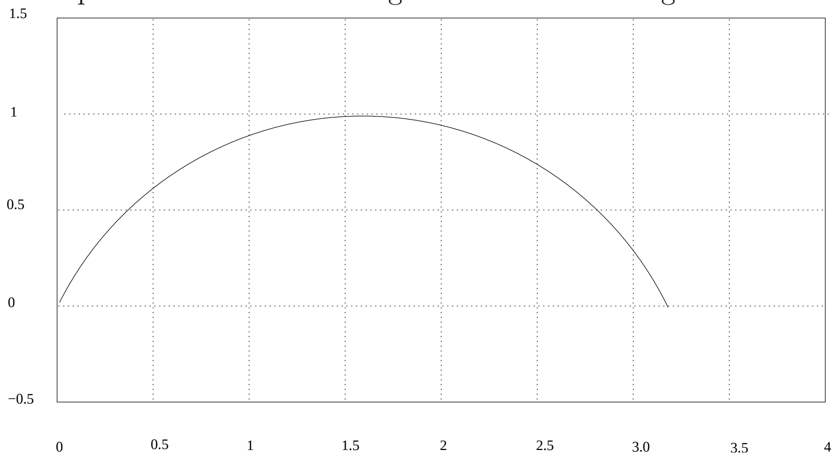
3. (20 points) We wish to solve the equation

$$\sin(x) = x^2$$

One solution is at $x = 0$, but we are interested in finding the other solution $x^\star \neq 0$

   (a) Write down an iteration formula for Newton's method for solving this problem, i.e an expression for x_k in terms of x_{k-1}.

(b) Consider the fixed point iteration $x_k = \sqrt{\sin(x_{k-1})}$ for the above equation. The graph below shows a plot of $g(x) = \sqrt{\sin(x)}$. Based on this graph, indicate approximately where the solution $x^\star$ is and explain (with adequate justification) whether or not you expect this fixed point iteration to converge to $x^\star$ if the initial guess is close enough.



4. (15 points) The matrix A below can be factored as $A = LU$ with L also given below

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 2 & 3 & 4 & 4 \\ 0 & 9 & 5 & 12 \\ -1 & 0 & 18 & 7 \end{bmatrix} \text{ and } L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ -1 & 0 & 4 & 1 \end{bmatrix}$$

(a) Determine the matrix U using the algorithm discussed in class. Show your work.

(b) Use forward and back substitution to solve the system $Ax = b$ for $b = [3 \quad 5 \quad 2 \quad 10]^T$.